

# Combinatorics and Numerical Analysis Joint Workshop

This workshop will be held with collaboration of Combinatorics Seminar and Numerical Analysis Seminar in Kyushu University.

## Organizers:

Yoshihiro Mizoguchi (Kyushu University),  
Tetsuji Taniguchi (Matsue College of Technology),  
Tsuyoshi Miezeki (Oita National College of Technology),  
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Advisary: Eiichi Bannai (Shanghai Jiao Tong University / Kyushu University)

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**Date:** February 17, 2012. 10:00–18:00

**Location:** Meeting Room, Nishijin Plaza, Kyushu University  
(2-16-23 Nishijin, Sawaraku, Fukuoka)

**URL:** <http://comb.math.kyushu-u.ac.jp/20120217/>

(Combinatorics) <http://comb.math.kyushu-u.ac.jp/>

(Numerical Analysis) <http://www2.math.kyushu-u.ac.jp/QNA/>

(GCOE) <http://gcoe-mi.jp/>

## Program

**10:00 – 11:00** Yaokun Wu (Shanghai Jiao Tong University)

Some combinatorial analysis of infinite matrix product

**11:30 – 12:10** Kenta Ozeki (National Institute of Informatics, Japan)

On the Hamiltonicity of graphs on a surface

**13:50 – 14:50** Xiaojun Chen (The Hong Kong Polytechnic University)

Computational Existence Proofs for Spherical  $t$ -Designs

**15:20 – 16:00** Takuma Kimura (Sasebo National College of Technology), Takehiko Kinoshita (RIMS, Kyoto University) and Mitsuhiro T. Nakao (Sasebo National College of Technology)

A numerical method to prove the existence of solutions for nonlinear parabolic problems

**16:20 – 17:00** Mamoru Tanaka (Tohoku University)

Higher eigenvalues of the Laplacian on a graph and partitions of the graph

**17:20 – 18:00** Mitsugu Hirasaka (Pusan National University)

Characterization of  $p$ -valenced association schemes

**19:00** – Banquet

## Abstract

吴耀琨 — Yaokun Wu

(上海交通大学 — Shanghai Jiao Tong University)

Title: Some combinatorial analysis of infinite matrix product

Abstract: The infinite product of a nonnegative square matrix is well understood thanks to the Perron-Frobenius Theory. In many contexts, say inhomogeneous Markov chain or opinion dynamics, one needs to consider the infinite product of several nonnegative square matrices of the same size. This general problem is much more complicated and seems that there is not any systematic theory for the analysis of the relevant dynamical behavior yet.

In this talk, we will discuss some combinatorial results obtained by the speaker and others (to be named during the lecture) on the dynamical behavior of the infinite matrix product of a set of matrices (of some special forms).

小関 健太 — Kenta Ozeki

(国立情報学研究所 — National Institute of Informatics, Japan)

Title: On the Hamiltonicity of graphs on a surface

Abstract: A cycle in a graph  $G$  is called Hamiltonian if it passes through all vertices in  $G$ . In this talk, we will concentrate on a Hamiltonian cycle in graphs on a Topological surface, for example, the sphere (the plane), the projective plane, the torus, and so on. One of the most classical result of this area is the one due to Tutte that states that “every 4-connected plane graph has a Hamiltonian cycle”. I would like to introduce some other results, some of which are obtained very recently. I also mention the connection between “the toughness” and the Hamiltonicity of graphs on a surface.

This is a joint work with K. Kawarabayashi (National Institute of Informatics, Japan).

陳小君 — Xiaojun Chen

(香港理工大学 — The Hong Kong Polytechnic University)

Title: Computational Existence Proofs for Spherical  $t$ -Designs

Abstract: Spherical  $t$ -designs provide quadrature rules for the sphere which are exact for polynomials up to degree  $t$ . In this talk, we propose a computational algorithm based on interval arithmetic which, for given  $t$ , upon successful completion will have proved the existence of a  $t$ -design with  $(t+1)^2$  nodes and will have computed narrow interval enclosures which are known to contain these nodes with mathematical certainty. Since there is no theoretical result which proves the existence of a  $t$ -design with  $(t+1)^2$  nodes for arbitrary  $t$ , our method contributes to the theory because it was tested successfully for  $t = 1, 2, \dots, 100$ , i.e., for all  $t$  considered so far. The  $t$ -design is usually not unique; our method aims at finding a well-conditioned one. The method relies on computing an interval enclosure for the zero of a highly non-linear system of dimension  $(t+1)^2$ . We therefore develop several special approaches which allow us to use interval arithmetic efficiently in this particular situation. The computations were all done using the MATLAB toolbox INTLAB. At the end of this talk, applications of well conditioned spherical designs for integration, interpolation and regularized least squares approximations on the two-sphere are discussed.

Joint work with Congpei An, Andreas Frommer, Bruno Lang, Ian Sloan and Womersley.

## References

- [1] C. An, X. Chen, I. H. Sloan and R. S. Womersley, Well conditioned spherical designs for integration and interpolation on the two-sphere, *SIAM J. Numerical Analysis*, 48(2010), 2135–2157.
- [2] C. An, X. Chen, I. H. Sloan and R. S. Womersley, Regularized least squares approximations on the sphere using spherical designs, submitted to *SIAM J. Numerical Analysis*, under revision.
- [3] X. Chen, A. Frommer and B. Lang, Computational existence proofs for spherical  $t$ -designs, *Numerische Mathematik*, 117(2011), 289–305.
- [4] X. Chen and R. S. Womersley, Existence of solutions to systems of underdetermined equations and spherical designs, *SIAM J. Numerical Analysis*, 44(2006), 2326–2341.
- [5] X. Chen, R. S. Womersley and J. J. Ye, Minimizing the condition number of a Gram matrix, *SIAM J. Optimization*, 21(2011), 127–148.

木村 拓馬 — Takuma Kimura <sup>†</sup>, 木下 武彦 — Takehiko Kinoshita <sup>‡</sup>,  
中尾 充宏 — Mitsuhiro T. Nakao <sup>†</sup>

(<sup>†</sup> 佐世保工業高等専門学校 — Sasebo National College of Technology, <sup>‡</sup> 京都大学数理解析研究所 — Research Institute for Mathematical Sciences, Kyoto University)

Title: A numerical method to prove the existence of solutions for nonlinear parabolic problems

Abstract: We present numerical verification methods for parabolic problems. Our main result is a constructive a posteriori estimates of inverse operators for initial-boundary value problems in linear parabolic PDEs on a bounded domain. The proposed a posteriori estimates is based on error analysis of the Galerkin approximation for boundary value problems in space direction and the piecewise linear interpolation for initial value problems in time. Applying the result, we can numerically prove the existence of solutions for nonlinear parabolic initial-boundary value problems. Some numerical results will be shown in the talk.

田中 守 — Mamoru Tanaka

(東北大学大学院 理学研究科 — Tohoku University)

Title: Higher eigenvalues of the Laplacian on a graph and partitions of the graph

Abstract: We can regard the 2-nd eigenvalue of the Laplacian on a connected finite graph as strength of connection between two disjoint subgraphs in the graph. In this talk, I will give a relation between the  $k$ -th eigenvalue of the Laplacian on a connected finite graph and the minimum among the 2-nd eigenvalues of the Laplacians on the subgraphs in a partition of the graph.

平坂 貢 — Mitsugu Hirasaka

(釜山国立大学 — Pusan National University)

Title: Characterization of  $p$ -valenced association schemes

Abstract: Let  $(X, \{R_i\}_{0 \leq i \leq d})$  be an association scheme and  $p$  a prime. We say that  $(X, \{R_i\}_{0 \leq i \leq d})$  is  $p$ -valenced if  $k_i$  is a power of  $p$  for each  $i$  with  $0 \leq i \leq d$  where  $k_i$  is the constant out-degree of the digraph  $(X, R_i)$ . In this talk we show some conditions for a  $p$ -valenced association scheme to be induced by a transitive permutation group.